Ejercicios y problemas

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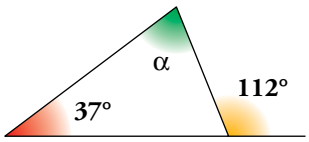
Practica

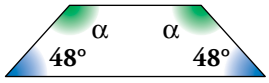
Ángulos

1. **Halla el valor del ángulo α en cada uno de estos casos:**

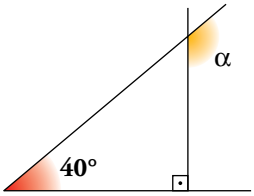
a)



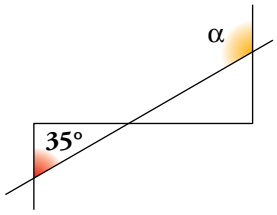
b)



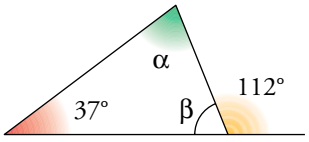
c)



d)



a)

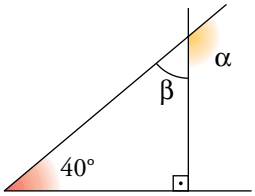


$\beta = 180° - 112° = 68°$

$\alpha = 180° - 37° - 68° = 75°$

b) $2\alpha = 360° - 48° \cdot 2 \rightarrow \alpha = 132°$

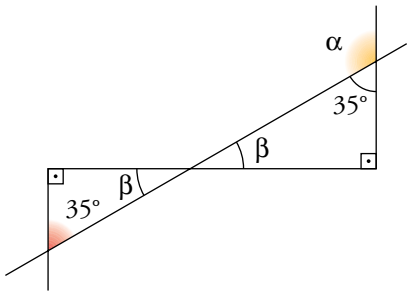
c)



$\beta = 180° - 90° - 40° = 50°$

$\alpha = 180° - 50° = 130°$

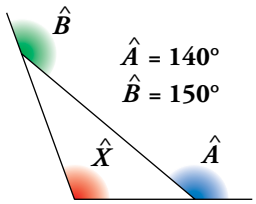
d)



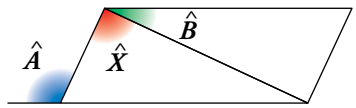
$\alpha = 180° - 35° = 145°$

2. **Calcula la medida de $\hat{X}$ en cada caso:**

a)



b)



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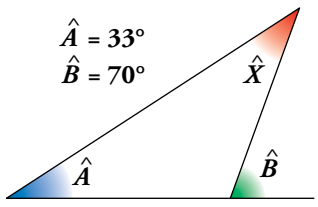
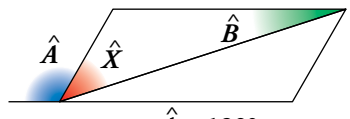


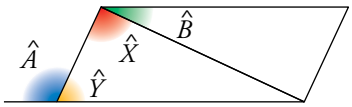
d)



a) $\hat{A} = 140° \rightarrow 180° - 140° = 40°$; $\hat{B} = 150° \rightarrow 180° - 150° = 30°$;

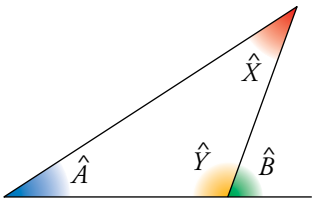
$\hat{X} = 180° - 40° - 30° = 110°$

b)



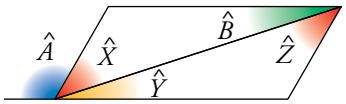
$\hat{Y} = 180° - 115° = 65°$; $\hat{Z} = 180° - 25° - 65° = 90°$; $\hat{X} = \hat{Z} = 90°$

c)



$\hat{Y} = 180° - 70° = 110°$; $\hat{X} = 180° - 110° - 33° = 37°$

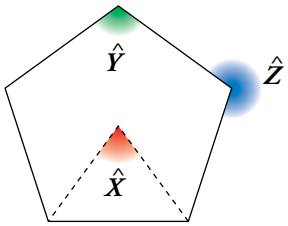
d)



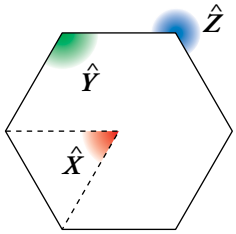
$\hat{X} + \hat{Y} = 180° - 120° = 60°$; $\hat{B} + \hat{Z} = 60° \rightarrow \hat{Z} = 60° - 18° = 42°$; $\hat{X} = \hat{Z} = 42°$

3. **Calcula los ángulos $\hat{X}$, $\hat{Y}$, $\hat{Z}$ en los siguientes polígonos regulares:**

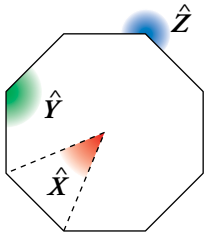
a)



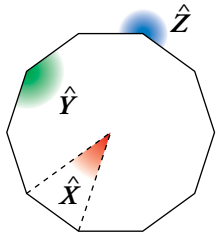
b)



c)

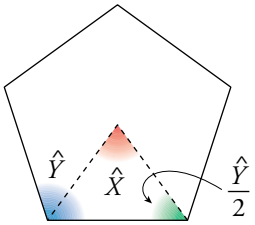


d)



a) $\hat{X}$ es un ángulo central del pentágono regular.

Por tanto, $\hat{X} = \dfrac{360°}{5} = 72°$.



$$\frac{\widehat{Y}}{2} + \frac{\widehat{Y}}{2} + \hat{X} = 180°$$

$\hat{Y} = 180° - \hat{X} = 180° - 72° = 108°$

$\hat{Z} = 360° - \hat{Y} = 360° - 108° = 252°$

b) $\hat{X} = 360° : 6 = 60°$

$\hat{Y} = \dfrac{(6-2)\cdot 180°}{6} = 4 \cdot 30° = 120°$

$\hat{Z} = 360° - 120° = 240°$

c) $\hat{X} = 360° : 8 = 45°$

$\hat{Y} = \dfrac{(8-2)\cdot 180°}{8} = 135°$

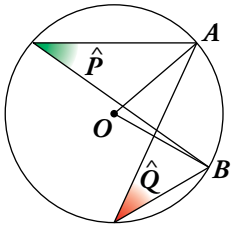
$\hat{Z} = 360° - 135° = 225°$

d) $\hat{X}$ es un ángulo central del decágono regular.

Por tanto, $\hat{X} = \dfrac{360°}{10} = 36°$.

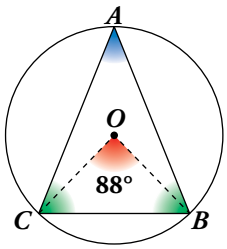
$\hat{Y} = \dfrac{180° \cdot (10-2)}{10} = 144°$; $\hat{Z} = 360° - 144° = 216°$

4. **Indica cuánto miden los ángulos $\hat{P}$ y $\hat{Q}$, sabiendo que $\widehat{AOB} = 70°$.**



$\hat{P} = \hat{Q} = \dfrac{70°}{2} = 35°$

5. **El triángulo ABC es isósceles. ¿Cuánto miden sus ángulos?**



$\hat{A}$ es un ángulo inscrito cuyo central correspondiente es $\widehat{BOC} = 88°$.

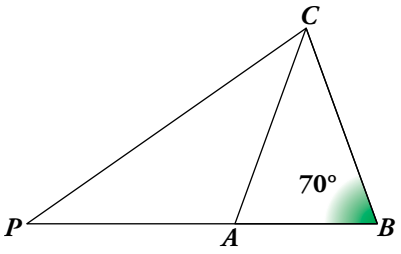
$\hat{A} = 88° : 2 = 44°$

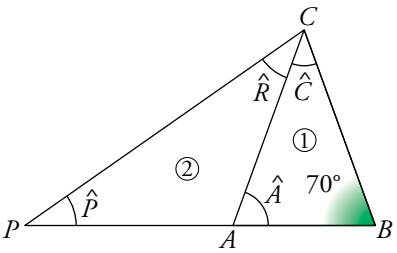
$\hat{A}$, $\hat{B}$ y $\hat{C}$ suman 180° y $\hat{B} = \hat{C}$.

$(180° - 44°) : 2 = 136° : 2 = 68°$

$\hat{A} = 44°$, $\hat{B} = \hat{C} = 68°$

6. **Sabiendo que $\overline{PA} = \overline{AC} = \overline{BC}$, $\hat{B} = 70°$, halla el ángulo $\widehat{PCB}$ en el siguiente triángulo:**





Si $\overline{AC} = \overline{BC}$, el triángulo ABC es isósceles, tiene dos ángulos iguales, $\hat{A} = \hat{B} = 70°$. El otro ángulo mide $\hat{C} = 180° - 2 \cdot 70° = 40°$.

Si $\overline{PA} = \overline{AC}$, el triángulo ACP es isósceles, tiene dos ángulos iguales, $\hat{P} = \hat{R}$. El otro ángulo mide $\hat{Q} = 180° - 70° = 110°$ y, por tanto, $\hat{R} = \dfrac{180° - 110°}{2} = 35°$.

Por último, $\widehat{PCB} = \hat{C} + \hat{R} = 40° + 35° = 75°$.

Semejanza

7. **Dos triángulos ABC y $A'B'C'$ son semejantes con razón de semejanza 1,2. Calcula los lados del triángulo $A'B'C'$ sabiendo que:**

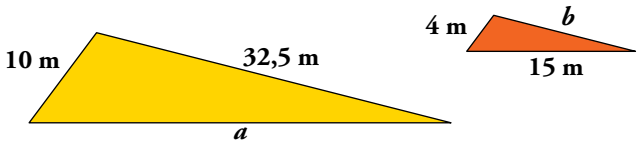
$\overline{AB}$ = 16 cm **$\overline{BC}$ = 25 cm** **$\overline{AC}$ = 39 cm**

$\overline{A'B'} = 1{,}2 \cdot 16 = 19{,}2$ cm

$\overline{B'C'} = 1{,}2 \cdot 25 = 30$ cm

$\overline{A'C'} = 1{,}2 \cdot 39 = 46{,}8$ cm

8. **Halla las longitudes de los lados a y b sabiendo que estos dos triángulos tienen sus lados paralelos:**



Como todos sus lados son paralelos, sus ángulos son iguales, por lo que los dos triángulos son semejantes. Así:

$$\frac{10}{4} = \frac{a}{15} = \frac{32{,}5}{b}$$

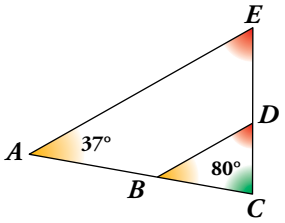
$$\frac{10}{4} = \frac{a}{15} \rightarrow 4a = 150 \rightarrow a = 37{,}5 \text{ m}$$

$$\frac{10}{4} = \frac{32{,}5}{b} \rightarrow 10b = 130 \rightarrow b = 13 \text{ m}$$

9. **Si BD es paralelo a AE, y $\overline{AC}$ = 15 cm, $\overline{CE}$ = 11 cm, $\overline{BD}$ = 6,4 cm, $\overline{AE}$ = 18 cm:**

a) Calcula $\overline{CD}$ y $\overline{BC}$.

b) Si $\hat{A}$ = 37° y $\hat{C}$ = 80°, halla $\hat{E}$, $\hat{B}$ y $\hat{D}$.



Por semejanza de triángulos:

a) $\frac{18}{6{,}4} = \frac{11}{\overline{CD}} \rightarrow \overline{CD} = \frac{11 \cdot 6{,}4}{18} \approx 3{,}9$ cm

$\frac{18}{6{,}4} = \frac{15}{\overline{BC}} \rightarrow \overline{BC} = \frac{15 \cdot 6{,}4}{18} \approx 5{,}33$ cm

b) $\hat{E} = 180° - 37° - 80° = 63°$

$\hat{B} = \hat{A} = 37°$

$\hat{D} = \hat{E} = 63°$

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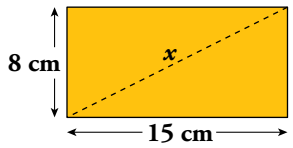
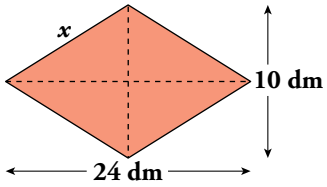
Teorema de Pitágoras

10. Calcula el valor de x en cada caso:

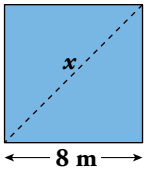
a)



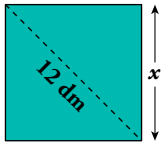
b)



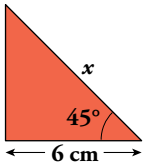
c)



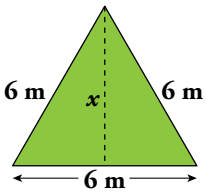
d)



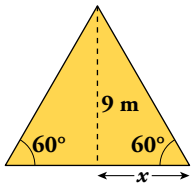
e)



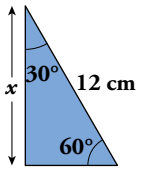
f)



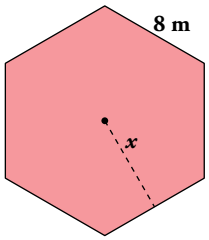
g)



h)



i)



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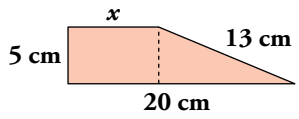
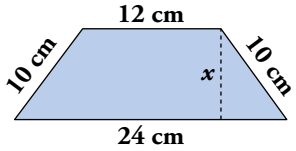
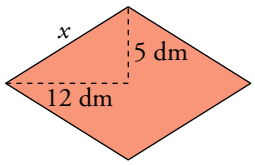


k)



a) $x = \sqrt{8^2 + 15^2} = \sqrt{289} = 17$ cm

b)



$x = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$ dm

c) $x = \sqrt{8^2 + 8^2} = \sqrt{128} \approx 11{,}3$ m

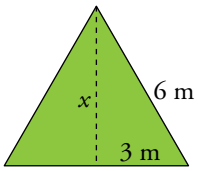
d) $x^2 + x^2 = 12^2 \rightarrow 2x^2 = 144 \rightarrow$

$\rightarrow x = \sqrt{72} \approx 8{,}5$ dm

e) Como es un triángulo rectángulo con un ángulo de 45°, el otro tendrá que medir 45° también, por lo que sabemos que el triángulo es isósceles. Así:

$x = \sqrt{6^2 + 6^2} = \sqrt{72} \approx 8{,}5$ cm

f)



$x = \sqrt{6^2 - 3^2} = \sqrt{27} \approx 5{,}2$ m